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PROPERTIES OF UNION ON CONSTANT INTERVAL VALUED INTUITIONISTIC FUZZY GRAPH

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Abstract: In this paper, some properties of union on constant interval valued intuitionistic fuzzy graph are defined, and also theorems related to these concepts are derived.

Keywords and Phrases: Interval valued intuitionistic fuzzy graph, constant IVIFG, Union, degree of fuzzy graph.

1. Introduction

Intuitionistic Fuzzy Set was introduced by K.T.Atanassov [2]. Research on the theory of Intuitionistic Fuzzy sets (IFS) has been witnessing an exponential growth in mathematics and its application. M.G.Karunambigai and R.Parvathi introduced intuitionistic fuzzy graph as a special case of Atanassov's IFG. S. Thilagavathi, R. parvathi, M.G. Karunambigai [5] introduced operations on intuitionistic fuzzy graph. K.Attannasov and G.Gargov [1] introduced the interval valued intuitionistic fuzzy set. Later S. Ismail Mohideen, A. Nagoor Gani, B. Fathima Kani and C.Yasmin [3] defined Regular Properties of operations on intuitionistic fuzzy graph.

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This leads to the consideration of the operation on interval valued intuitionistic fuzzy graph (IVIFG). In this paper properties of union on constant interval valued intuitionistic fuzzy graph are established.

2. Basic definition

H= (τ, ρ) is called a fuzzy subgraph of a fuzzy graph G = (σ, μ) if $\tau(u) \leq \sigma(u)$, for all $u \in V$ and $\rho(u, v) \leq \mu(u, v)$ for all $u, v \in V$. The underlying crisp graph of a fuzzy graph G = (σ, μ) is denoted by $G^*: (\sigma^*, \mu^*)$, where

$\sigma^* = \{ u \in V / \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in V \times V / \mu(u, v) > 0\}$. A fuzzy graph G= (σ, μ) is a strong fuzzy graph if $\mu(u, v) = \sigma(u) \wedge \sigma(u)$ for all $(u, v) \in \mu^*$.A fuzzy graph G= (σ, μ) is a complete fuzzy graph if $\mu(u, v) = \sigma(u) \wedge \sigma(u)$ for all $u, v \in \sigma^*$. Let G = (σ, μ) be a fuzzy graph G^* if d_G(v) = K, for all $v \in V$. That is if each vertex has same degree K, then G is said to be a regular fuzzy graph of degree K (or) a K-regular fuzzy graph. The degree of a vertex u of a fuzzy graph G is defined by d(u) =$\sum_{u \neq v} \mu(u, v). \forall\, v \in V$

An intuitionistic fuzzy graph (IFG) is of the form G:(V,E) where

i) The function $\mu_1: V \to [0,1]$ & $\gamma_1: V \to [0,1]$ denote the degree of membership and nonmembership of the element $v_i \in V$ respectively. Such that $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V$, (i= 1,2,….n).

ii) The function $\mu_2: V \times V \to [0,1]$& $\gamma_2: V \times V \to [0,1]$ are defined by

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$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$

$\gamma_2(v_i, v_j) \leq \max[\gamma_1(v_i), \gamma_1(v_j)]$.

An intuitionistic fuzzy graph G is said to be strong IFG, if $\mu_{2ij} = \min(\mu_1(v_i), \mu_1(v_j))$ and $\gamma_{2ij} = \max(\gamma_1(v_i), \gamma_1(v_j))$ for every $(v_i, v_j) \in E$. An IFG G =(V,E) is said to be a complete IFG if $\mu_{2ij} = \min(\mu_1(v_i), \mu_1(v_j))$ and $\gamma_{2ij} = \max(\gamma_1(v_i), \gamma_1(v_j))$ for every $(v_i, v_j) \in V$. let G = $\{(\mu_1, \gamma_1), (\mu_2, \gamma_2)\}$ be an IFG. Then the degree of a vertex v_i is defined as $d(v_i) = \left[\sum_{(v_i,v_j)\in E} \mu_2\,(v_i, v_j); \sum_{(v_i,v_j)\in E} \gamma_2\,(v_i, v_j)\right]$. Let G = {V = $(\mu_1,\gamma_1), E = (\mu_2, \gamma_2)\}$ be an IFG. Then the order of G is defined to be O(G) = ($(O_\mu(G), O_\gamma(G))$ where $O_\mu(G) = \sum_{v\in V} \mu_1\,(v)$ and $O_\gamma(G) = \sum_{v\in V} \gamma_1\,(v)$. Let G = $((\mu_1,\gamma_1)\ (\mu_{2\,,}\gamma_2))$ be an intuitionistic fuzzy graph on $G^* = (V, E\,)$ .If d_μ (v) = K_1 and $d_\gamma(v) = K_2, \forall\, v \in V$,then the graph is called constant intuitionistic fuzzy graph of degree (K_1, K_2) or (K_1, K_2) constant intutionistic fuzzy graph.

An interval valued intuitionistic fuzzy graph (IVIFG) with underlying set V is defined to be a pair G = (μ, γ), where

$i)$ $\mu_1: V \to D[0,1]$ and $\gamma_1: V \to D[0,1]$ denote the degree of membership and membership of the element $v_i \in V$,respectively and $0 \leq \mu_1^R(v_i) + \gamma_1^R(v_i) \leq 1$ for every $v_i \in V$,(i=1,2,….n),

ii) The function $\mu_2: E \subseteq V \times V \to D[0,1]$ and $\gamma_2: E \subseteq V \times V \to D[0,1]$ where

$\mu_2^L(v_i, v_j) \leq \min[\mu_1^L(v_i), \mu_1^L(v_j)]$ and $\mu_2^R(v_i, v_j) \leq \min[\mu_1^R(v_i), \mu_1^R(v_j)]$

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$\gamma_2^L(v_i,v_j) \leq \max[\gamma_1^L(v_i),\gamma_1^L(v_j)]$and $\gamma_2^R(v_i,v_j) \leq \max[\gamma_1^R(v_i),\gamma_1^R(v_j)]$ such that

$0 \leq \mu_2^R(v_i,v_j) + \gamma_2^R(v_i,v_j) \leq 1$ for every $(v_i,v_j) \in E$, i, j =1,2,…n.

Let G be an IVIFG, if $d_\mu(v_i) = (K_i,K_j)$ and $d_\gamma(v_i) = (K_i,K_j)$ for all $v_i \in V$, then the graph G is constant IVIFG with degree $[(K_i,K_j),(K_i,K_j)]$.

3. Properties of Union on constant interval valued intuitionistic fuzzy graph

3.1. Union on Interval Valued Intuitionistic Fuzzy Graph:

Let $G_1:(V_1,E_1)$ and $G_2:(V_2,E_2)$ be two interval valued intuitionistic fuzzy graph with $V_1 \cap V_2 \neq \phi$. Then the union of interval valued intuitionistic fuzzy graph G_1 and G_2 is an interval valued intuitionistic fuzzy graph G = $G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$ with the condition that

a) $(\mu_{A1}^L \cup \mu_{A2}^L)(u) = \mu_{A1}^L(u) if\ u \in V_1 - V_2$

$(\mu_{A1}^L \cup \mu_{A2}^L)(u) = \mu_{A2}^L(u) if\ u \in V_2 - V_1$

$(\mu_{A1}^L \cup \mu_{A2}^L)(u) = max(\mu_{A1}^L(u),\mu_{A2}^L(u))\ if\ u \in V_1 \cap V_2$

b) $(\mu_{A1}^R \cup \mu_{A2}^R)(u) = \mu_{A1}^R(u) if\ u \in V_1 - V_2$

$(\mu_{A1}^R \cup \mu_{A2}^R)(u) = \mu_{A2}^R(u) if\ u \in V_2 - V_1$

$(\mu_{A1}^R \cup \mu_{A2}^R)(u) = max(\mu_{A1}^R(u),\mu_{A2}^R(u))\ if\ u \in V_1 \cap V_2$

c) $(\gamma_{A1}^L \cup \gamma_{A2}^L)(u) = \gamma_{A1}^L(u) if\ u \in V_1 - V_2$

$(\gamma_{A1}^{L} \cup \gamma_{A2}^{L})(u) = \gamma_{A2}^{L}(u) if\ u \in V_2 - V_1$

$(\gamma_{A1}^{L} \cup \gamma_{A2}^{L})(u) = min(\gamma_{A1}^{L}(u), \gamma_{A2}^{L}(u)) if\ u \in V_1 \cap V_2$

d) $(\gamma_{A1}^{R} \cup \gamma_{A2}^{R})(u) = \gamma_{A1}^{R}(u) if\ u \in V_1 - V_2$

$(\gamma_{A1}^{R} \cup \gamma_{A2}^{R})(u) = \gamma_{A2}^{R}(u) if\ u \in V_2 - V_1$

$(\gamma_{A1}^{R} \cup \gamma_{A2}^{R})(u) = min(\gamma_{A1}^{R}(u), \gamma_{A2}^{R}(u)) if\ u \in V_1 \cap V_2$

e) $(\mu_{B1}^{L} \cup \mu_{B2}^{L})(u_i u_j) = \mu_{B1}^{L}(u_i u_j) if\ u_i u_j \in E_1 - E_2$

$(\mu_{B1}^{L} \cup \mu_{B2}^{L})(u_i u_j) = \mu_{B2}^{L}(u_i u_j) if\ u_i u_j \in E_2 - E_1$

$(\mu_{B1}^{L} \cup \mu_{B2}^{L})(u_i u_j) = \max\left(\mu_{B1}^{L}(u_i u_j), \mu_{B2}^{L}(u_i u_j)\right) if\ u_i u_j \in E_1 \cap E_2$

f) $(\mu_{B1}^{R} \cup \mu_{B2}^{R})(u_i u_j) = \mu_{B1}^{R}(u_i u_j) if\ u_i u_j \in E_1 - E_2$

$(\mu_{B1}^{R} \cup \mu_{B2}^{R})(u_i u_j) = \mu_{B2}^{R}(u_i u_j) if\ u_i u_j \in E_2 - E_1$

$(\mu_{B1}^{R} \cup \mu_{B2}^{R})(u_i u_j) = \max\left(\mu_{B1}^{R}(u_i u_j), \mu_{B2}^{R}(u_i u_j)\right) if\ u_i u_j \in E_1 \cap E_2$

g) $(\gamma_{B1}^{L} \cup \gamma_{B2}^{L})(u_i u_j) = \gamma_{B1}^{L}(u_i u_j) if\ u_i u_j \in E_1 - E_2$

$(\gamma_{B1}^{L} \cup \gamma_{B2}^{L})(u_i u_j) = \gamma_{B2}^{L}(u_i u_j) if\ u_i u_j \in E_2 - E_1$

$(\gamma_{B1}^{L} \cup \gamma_{B2}^{L})(u_i u_j) = \min\left(\gamma_{B1}^{L}(u_i u_j), \gamma_{B2}^{L}(u_i u_j)\right) if\ u_i u_j \in E_1 \cap E_2$

h) $(\gamma_{B1}^{R} \cup \gamma_{B2}^{R})(u_i u_j) = \gamma_{B1}^{R}(u_i u_j) if\ u_i u_j \in E_1 - E_2$

$(\gamma_{B1}^{R} \cup \gamma_{B2}^{R})(u_i u_j) = \gamma_{B2}^{R}(u_i u_j) if\ u_i u_j \in E_2 - E_1$

$(\gamma_{B1}^{R} \cup \gamma_{B2}^{R})(u_i u_j) = \min\left(\gamma_{B1}^{R}(u_i u_j), \gamma_{B2}^{R}(u_i u_j)\right) if\ u_i u_j \in E_1 \cap E_2$

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3.2. Properties of Union on Constant Interval Valued Intuitionistic Fuzzy Graph:

Theorem 3.2.1:

Let G_1 and G_2 be two interval valued intuitionistic fuzzy graph such that $V_1 \cap V_2 = \phi$. Then $G_1 \cup G_2$ is a $[(K_1, K_2), (K_1, K_2)]$ constant interval valued intuitionistic fuzzy graph iff G_1 and G_2 are $[(K_1, K_2), (K_1, K_2)]$ constant interval valued intuitionistic fuzzy graph.

Proof:

Since $V_1 \cap V_2 = \phi$

$$d_{G_1 \cup G_2}(u) = \begin{cases} d_{G_1}(u) \; if \; u \in V_1 \\ d_{G_2}(u) \; if \; u \in V_2 \end{cases}$$

$$d_{G_1 \cup G_2}(u) = [(K_1, K_2), (K_1, K_2)] \Leftrightarrow d_{G_1}(u) = [(K_1, K_2), (K_1, K_2)] \text{ and}$$

$$d_{G_2}(u) = [(K_1, K_2), (K_1, K_2)]$$

Hence $G_1 \cup G_2$ is a $[(K_1, K_2), (K_1, K_2)]$ constant interval valued intuitionistic fuzzy graph iff G_1 and G_2 are $[(K_1, K_2), (K_1, K_2)]$ constant interval valued intuitionistic fuzzy graphs.

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Theorem 3.2.2:

Let G_1 be interval valued intuitionistic fuzzy subgraph of G_2. Then $G_1 \cup G_2$ is a $[(K_1, K_2), (K_1, K_2)]$ constant interval valued intuitionistic fuzzy graph iff G_2 is a $[(K_1, K_2), (K_1, K_2)]$ constant interval valued intuitionistic fuzzy graphs.

Proof:

Let $G_1 = [(\mu_1^L, \mu_1^R)(\gamma_1^L, \gamma_1^R), (\mu_2^L, \mu_2^R)(\gamma_2^L, \gamma_2^R)]$ be an interval valued intuitonistic fuzzy subgraph of $G_2 = [(\mu_1^{'L}, \mu_1^{'R})(\gamma_1^{'L}, \gamma_1^{'R}), (\mu_2^{'L}, \mu_2^{'R})(\gamma_2^{'L}, \gamma_2^{'R})]$

Then $\mu_1^L \leq \mu_1^{'L}$, $\mu_1^R \leq \mu_1^{'R}$, $\gamma_1^L \geq \gamma_1^{'L}$, $\gamma_1^R \geq \gamma_1^{'R}$

$\mu_2^L \leq \mu_2^{'L}$, $\mu_2^R \leq \mu_2^{'R}$, $\gamma_2^L \geq \gamma_2^{'L}$, $\gamma_2^R \geq \gamma_2^{'R}$

Therefore $\mu_1^L \cup \mu_1^{'L} = \mu_1^{'L}$ $\gamma_1^L \cup \gamma_1^{'L} = \gamma_1^{'L}$

$\mu_1^R \cup \mu_1^{'R} = \mu_1^{'R}$ $\gamma_1^R \cup \gamma_1^{'R} = \gamma_1^{'R}$

$\mu_2^L \cup \mu_2^{'L} = \mu_2^{'L}$ $\gamma_2^L \cup \gamma_2^{'L} = \gamma_2^{'L}$

$\mu_2^R \cup \mu_2^{'R} = \mu_2^{'R}$ $\gamma_2^R \cup \gamma_2^{'R} = \gamma_2^{'R}$

Hence $G_1 \cup G_2 = [(\mu_1^{'L}, \mu_1^{'R})(\gamma_1^{'L}, \gamma_1^{'R}), (\mu_2^{'L}, \mu_2^{'R})(\gamma_2^{'L}, \gamma_2^{'R})] = G_2$

That is $G_1 \cup G_2$ is a $[(K_1, K_2), (K_1, K_2)]$ constant interval valued intuitionistic fuzzy graph iff G_2 is a $[(K_1, K_2), (K_1, K_2)]$ constant interval valued intuitionistic fuzzy graphs.

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Theorem 3.2.3

Let G_1 and G_2 be two interval valued intuitionistic fuzzy graph such that $V_1 \cap V_2 = \phi$. If G_1 and G_2 are constant interval valued intuitionistic fuzzy graphs then $G_1 \cup G_2$ is not constant interval valued intuitionistic fuzzy graphs.

Proof:

Since $V_1 \cap V_2 = \phi$

$$d_{G_1 \cup G_2}(u) = \begin{cases} d_{G_1}(u) \; if \; u \in V_1 - V_2 \\ d_{G_2}(u) \; if \; u \in V_2 - V_1 \end{cases}$$

$$\text{So, } d_{G_1 \cup G_2}(u) = \begin{cases} [(K_1, K_2), (K_1, K_2)] \; if \; u \in V_1 - V_2 \\ [(K_3, K_4), (K_3, K_4)] \; if \; u \in V_2 - V_1 \end{cases}$$

That is $d_{G_1}(u) = [(K_1, K_2), (K_1, K_2)]$ for all $u \in V_1$

$d_{G_2}(u) = [(K_3, K_4), (K_3, K_4)]$ for all $u \in V_2$

Hence $G_1 \cup G_2$ is not constant interval valued intuitionistic fuzzy graphs.

Theorem 3.2.4

Let G_1 and G_2 be two interval valued intuitionistic fuzzy graph such that $V_1 \cap V_2 \neq \phi$. If G_1 and G_2 are constant interval valued intuitionistic fuzzy graphs then $G_1 \cup G_2$ is not constant interval valued intuitionistic fuzzy graphs.

Proof:

Since $V_1 \cap V_2 \neq \phi$

Given G_1 and G_2 are constant interval valued intuitionistic fuzzy graphs,

That is $d_{G_1}(u) = [(K_1, K_2), (K_1, K_2)]$ for all $u \in V_1$

$d_{G_2}(u) = [(K_3, K_4), (K_3, K_4)]$ for all $u \in V_2$

We know that $d_{G_1 \cup G_2}(u) = \begin{cases} d_{G_1}(u) \text{ } if \text{ } u \in V_1 - V_2 \\ d_{G_2}(u) \text{ } if \text{ } u \in V_2 - V_1 \\ d_{G_1}(u) + d_{G_2}(u) \text{ } if \text{ } u \in V_1 \cap V_2 \end{cases}$

$d_{G_1 \cup G_2}(u) = d_{G_1}(u)$

$= [(K_1, K_2), (K_1, K_2)]$ for all $u \in V_1 - V_2$

$d_{G_1 \cup G_2}(u) = d_{G_1}(u)$

$= [(K_3, K_4), (K_3, K_4)]$ for all $u \in V_2 - V_1$

Suppose, if there is a vertex $v \in V_1 \cap V_2$ such that

$d_{G_1 \cup G_2}(v) = d_{G_1}(v) + d_{G_2}(v)$

$= [(K_1, K_2), (K_1, K_2)] + [(K_3, K_4), (K_3, K_4)]$

$= [(K_1, K_2) + (K_3, K_4)]\ , [(K_1, K_2) + (K_3, K_4)]$

Hence $d_{G_1 \cup G_2}(u) \neq d_{G_1 \cup G_2}(v)$

Therefore $G_1 \cup G_2$ is not constant interval valued intuitionistic fuzzy graphs.

Conclusion:

In this paper, some new properties of union on constant interval valued intuitionistic fuzzy graph are discussed. It will be more useful for doing further research in the field of constant interval valued IFG.

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